18. Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

19. Verify divergence theorem for $F = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

20. Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x dx$.

Shanmuga Industries Arts & Science College T.V.Malai

APRIL/MAY 2025

FAEL22/CAEL22 — BASIC MATHEMATICS — II (ALLIED)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find the n^{th} derivative of $y = \sin(ax + b)$.

2. Find the maxima of the function $2x^3 - 3x^2 - 36x + 10$.

3. Solve $(D^3 - 3D^2 + 4)y = 0$.

4. Find the particular integral of $(D^2 - D - 2)y = 4e^{3x}$.

5. Eliminate a and b from $ax^2 + by^2 + z^2 = 1$.

6. Find the complete integral of $z = px + qy + pq$.

7. Find the unit normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2).

8. State Gauss Divergence theorem.

9. Evaluate $\int x \sin zx dx$.

10. Find $\int_0^{\frac{\pi}{2}} \sin^7 x dx$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Find y_n when $y = \frac{1}{x^2 + a^2}$.

Or

(b) If $y = \sin\left(m \sin^{-1} x\right)$ prove that $\left(1-x^2\right)y_2 - xy_1 + m^2 y = 0$ and $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$.

12. (a) Solve $\left(D^2 - 8D + 9\right)y = 8 \sin 5x$.

Or

(b) Solve $\left(D^3 - 2D + 4\right)y = e^x \cos x$.

13. (a) Solve $q = xp + p^2$.

Or

(b) Solve $q(p - \sin x) = \cos y$.

14. (a) If $\overline{A} = x^2 y\vec{i} - 2y^3 z^2 \vec{j} + xy^2 z\vec{k}$, find $div\overline{A}$ at $(1, -1, 1)$.

Or

(b) Show that

$\vec{F} = \left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}$ is irrotational. Find ϕ such that $F = \nabla\phi$.

15. (a) Evaluate $\int e^x \frac{x+1}{(x+2)^2} dx$.

Or

(b) Expresses $f(x) = x \ (-\pi < x < \pi)$ as a Fourier series with period 2π.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the n^{th} differential co-efficient of $\cos x \cos 2x \cos 3x$.

17. Solve $\left(D^3 - D^2 - D + 1\right)y = 1 + x^2$.

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